

## Resolving power of a Spectroscope and a Microscope.

Resolving power of a Spectroscope: — The resolving power  $R$  of a Spectroscope is defined as the ratio of the wavelength  $\lambda$  to the smallest change in wavelength  $d\lambda$ , that can be resolved.

$$\text{Thus } R = \frac{\lambda}{d\lambda}$$

If  $d\theta$  be the smallest angle that can be resolved in the image space

We may put

$$d\lambda = \frac{d\theta}{D}$$

Where  $D$  is dispersive power of the Spectroscope and is defined as the rate of variation of deviation with wavelength ( $D = \frac{d\theta}{d\lambda}$ )

$$\therefore R = D \frac{\lambda}{d\theta}$$

As we have shown above for a rectangular aperture of width  $e$ , we have

$$d\theta = \frac{\lambda}{e}$$

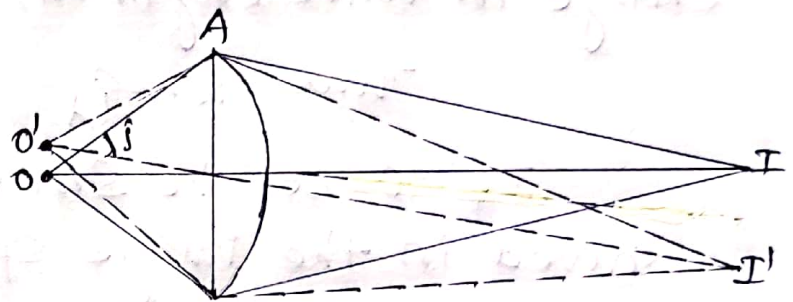
Hence the above expression becomes

$$R = D e$$

This is required formula of resolving power of a Spectroscope.

Resolving power of a Microscope: — The resolving power of a microscope represents its ability to form distinctly separate images of two objects lying close together. It is measured by the smallest distance between two point-objects whose images are just resolved by the objective of the microscope. The smaller is the distance, the higher is said to be the Resolving power.

Let  $o$  and  $o'$  be two point-objects whose images are just resolved by the



objective AB of a microscope. Let  $i$  be the semi vertical angle of the cone of rays received by the objective from  $o$ .

The boundary of the objective acts as a circular aperture. Hence the images of  $o$  and  $o'$  formed by the objective are actually Fraunhofer diffraction patterns. Each pattern consists of a central bright disc surrounded by a series of alternate dark and bright rings. The centres of the discs lie at  $I$  and  $I'$  the geometrical images of  $o$  and  $o'$  respectively.

According to criterion  $o$  and  $o'$  will be just resolved when the centre  $I$  of the disc due to  $o$  falls on the first dark ring



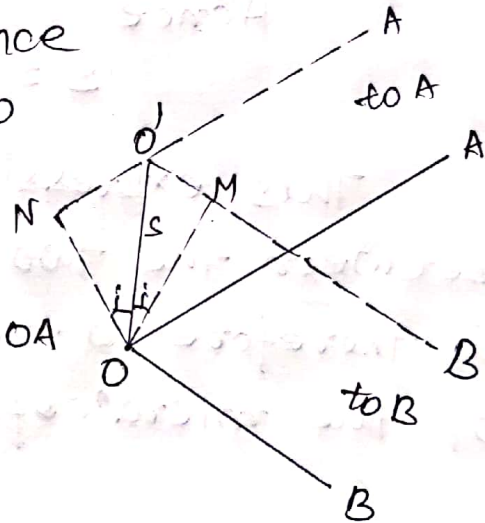
due to  $O'$  and vice-versa. This means that for just resolving, the waves from  $O'$ , after diffraction by the objective, must form the first dark ring passing through  $I$ . Airy has shown that this will happen the path difference between the extreme rays,  $O'B - O'A$  is given by

$$O'B - O'A = 1.22\lambda$$

Since the path  $AI$  and  $BI$  are equal, the above condition becomes

$$O'B - O'A = 1.22\lambda \quad \text{--- (1)}$$

Let  $s$  be the distance between  $O$  and  $O'$ . As  $O$  and  $O'$  are very close together, we can take  $O'A$  to be parallel to  $OA$  and  $O'B$ , parallel to  $OB$



from the figure

$$O'B - OB = O'M = s \sin i$$

$$\text{and } OA - O'A = O'N = s \sin i$$

Adding these equations and remembering that  $OA = OB$ , we get

$$O'B - O'A = 2s \sin i$$

Substituting this value of  $O'B - O'A$  in eqn (1)

$$2s \sin i = 1.22\lambda$$

$$\text{or } s = \frac{1.22\lambda}{2 \sin i}$$

If the space between the object and the objective is filled with an oil of refractive index  $\mu$ , then

$$S = \frac{1.22\lambda}{2\mu \sin i}$$

because the path difference  $O'B - O'A$  is then multiplied by  $\mu$ .

The quantity  $\mu \sin i$  is called the numerical aperture (NA) of the objective

Hence

$$S = \frac{1.22\lambda}{2 \cdot \text{N.A}}$$

This expression gives the linear distance between the two objects just resolved and is therefore, a measure of the resolving power of the microscope.